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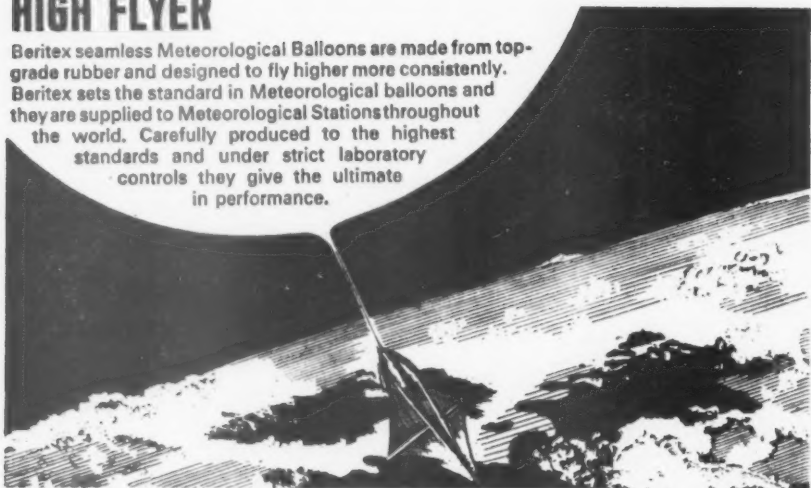
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A NOTE ON THE CALCULATION OF 'CUT-OFF' MACH NUMBER

By J. M. NICHOLLS

Summary. The paper gives a theoretical derivation of the Mach number which an aircraft must reach in order that any sonic bang produced will just reach the ground ('cut-off' Mach number). The derived formula is compared with a derivation, used in other literature, which was based on an incorrect assumption. A method of calculation of cut-off Mach number is presented together with an assessment of the probable errors in the calculated value.

Introduction. The Mach number, M , of an aircraft is defined as the ratio of its airspeed, V , to the local sound speed, a_A (suffix A denotes values at the aircraft); if the aircraft is travelling supersonically (i.e. $V \geq a_A$) then $M \geq 1$ and a shock wave (whose formation will be described in the next section) will be propagated from the aircraft but will not necessarily reach the ground because the path depends on atmospheric refraction. The 'cut-off' Mach number, M_c , is defined as the value of M which the aircraft must reach at any altitude in order that any shock wave produced at that altitude will just reach the ground. A 'sonic bang' is the noise which is heard as the shock wave passes the ear of an observer. If M_c varies markedly with height it may be possible during the transonic acceleration phase for aircraft to fly at greater speeds at some heights than at others without the shock wave reaching the ground. Airline operators may therefore require forecasts of the relationship between M_c and height. The purpose of this paper is threefold, namely (i) to familiarize forecasters with the concept of cut-off Mach number, (ii) to develop a formula from which M_c can be calculated in any atmosphere, elaborating on and correcting some previous work¹ on the subject and (iii) to deduce a reasonably accurate method of calculating M_c by hand.

Formulae for calculating M_c .

(i) *Level flight.* A generalized description of sonic bang formation and propagation can be found in other literature (e.g. Nicholls²), and only the propagation geometry which is needed to describe the subject of this paper is reproduced here.

Consider an aircraft travelling with constant supersonic velocity V in a still, isothermal atmosphere. For each point on the flight track a single acoustic

compression wave originating at the instantaneous aircraft position can be imagined, as shown in Figure 1. After a time T the waves originating at times t_0, t_1, \dots, T will be spheres of radius $a(T-t_0), a(T-t_1), \dots, 0$, where a is the speed of sound. The individual waves have an envelope where reinforcement takes place to give a shock wave, and in the atmosphere under consideration it can be seen that the reinforcement takes place along lines of propagation which are normal to the wavefront; the set of these lines (or wavenormals) from any point on the flight track forms a 3-dimensional cone known as a 'bang cone', which has an apex half-angle of $\cos^{-1}(1/M)$. The wavefront of the shock is also a cone known as the 'Mach cone', whose apex travels with the aircraft, with the apex half-angle $\sin^{-1}(1/M)$.

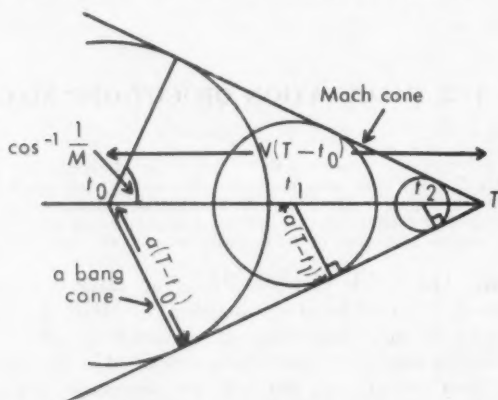


FIGURE 1—LOCATION OF MACH CONE AND COMPRESSION WAVES AT TIME T

a Speed of sound V Aircraft airspeed
 t_0, t_1, t_2, \dots, T Times at which the aircraft is at points shown

In an atmosphere with vertical variations of temperature and horizontal wind present, it can be shown (utilizing an axial system which moves with the wind at aircraft altitude) that the initial wavenormals still form a cone whose apex half-angle remains equal to $\cos^{-1}(1/M)$. Consider a co-ordinate system (x, y, z) , as shown in Figure 2, with origin at the instantaneous aircraft position and whose x -axis lies along the projection of a wavenormal on the horizontal plane and whose z -axis is vertical; the x -direction cosine, l_A , of the wavenormal at the point of origin is given by

$$l_A = \frac{1}{M \cos \theta}, \quad \dots (1)$$

where θ is the angle between the x -axis and the airspeed vector. The sign convention for θ is shown in Figure 3. Figure 4 shows how the wavefront is propagated at any level in the atmosphere beneath the aircraft, for which a is the sound speed and u is the wind component in the x -direction; l is the x -direction cosine of the wavenormal. In the non-stationary atmosphere under consideration the path of the wavefront is not described by the normals to the wavefront, but by 'rays', a 'ray' being defined as the path followed by an element of wavefront from the point of origin. As shown by Milne³ the

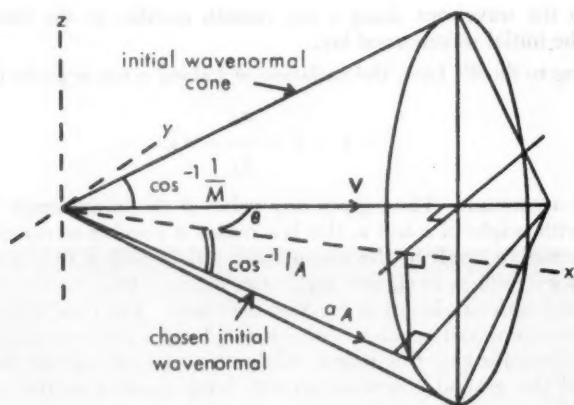


FIGURE 2—INITIAL WAVENORMAL GEOMETRY, LEVEL FLIGHT

a_A Speed of sound at aircraft

V Airspeed vector

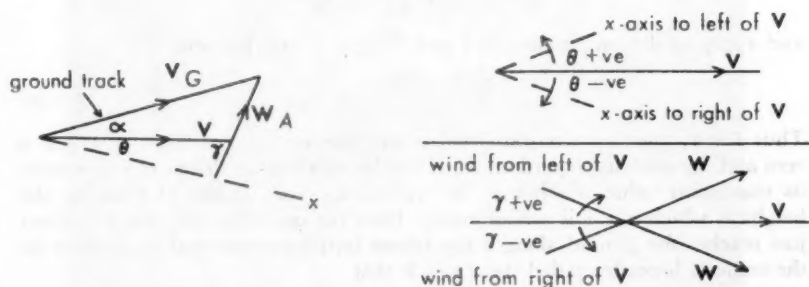


FIGURE 3—RELATIONSHIP BETWEEN AIRSPEED AND GROUNDSPD, AND SIGN CONVENTIONS

V Airspeed vector

W Wind vector
 A = Aircraft level

V_G Groundspeed vector

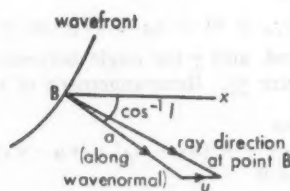


FIGURE 4—WAVEFRONT PROPAGATION AT ANY LEVEL BENEATH AIRCRAFT

a Speed of sound

u Wind component along x direction

l x -direction cosine of the wavenormal

normals to the wavefront along a ray remain parallel to the vertical plane in which the initial wavenormal lay.

According to Snell's Law, the variation of l along a ray is given by

$$\frac{a}{l} + u = C = \frac{a_A}{l_A} + u_A, \quad \dots (2)$$

where C is a constant. Thus, given any value of the initial angle θ , and the variation with height of a and u , this law makes it possible to calculate values of l at descending levels in the atmosphere. If, on doing this, it was found that $l > 1$ for a certain level, the physical meaning is that the ray would have been reflected upwards before it reached that level. The condition, therefore, that the wavefront just reaches the ground along a ray whose wavenormal was initially specified by θ is that a maximum value of l occurs between the aircraft and the ground (for wavenormals lying parallel to the x, z plane) and that this value is unity; in mathematical terms, the conditions are that $\partial l / \partial z = 0$ for $l = 1$, and l is never greater than unity. If equation (2) is differentiated with respect to z , then

$$\frac{1}{l} \frac{\partial a}{\partial z} - \frac{a}{l^2} \frac{\partial l}{\partial z} + \frac{\partial u}{\partial z} = 0, \quad \dots (3)$$

and applying the conditions $l = 1$ and $\partial l / \partial z = 0$, this becomes

$$\frac{\partial(a + u)}{\partial z} = 0. \quad \dots (4)$$

Thus $l = 1$, $\partial l / \partial z = 0$ at the level where the vertical gradient of $a + u$ is zero and, by studying equation (2), it can be seen that at this level $a + u$ takes its maximum value which may be written $a_H + u_H$, suffix H denoting the height at which $a + u$ is a maximum. Then the condition that the wavefront just reaches the ground along a ray whose initial wavenormal is specified by the angle θ , hereafter called the θ ray, is that

$$\frac{a_A}{l_A} + u_A = a_H + u_H \quad \dots (5)$$

since $l_H = 1$. Substituting for l_A from equation (1) and expanding the wind components u in terms of wind speed and direction, we find that the airspeed V_θ which the aircraft must reach before the wavefront reaches the ground along the θ ray is given by

$$V_\theta \cos \theta + W_A \cos(\gamma_A + \theta) = a_H + W_H \cos(\gamma_H + \theta), \quad \dots (6)$$

where W is the wind speed, and γ the angle between the airspeed and wind-velocity vectors (see Figure 3). Rearrangement of the last equation gives:

$$V_\theta = \frac{a_H}{\cos \theta} + (u_H - u_A) - (v_H - v_A) \tan \theta, \quad \dots (7)$$

where u and v are the components of wind speed along and normal to the airspeed vector. Further development of the theory varies according to the way in which H is dependent on θ .

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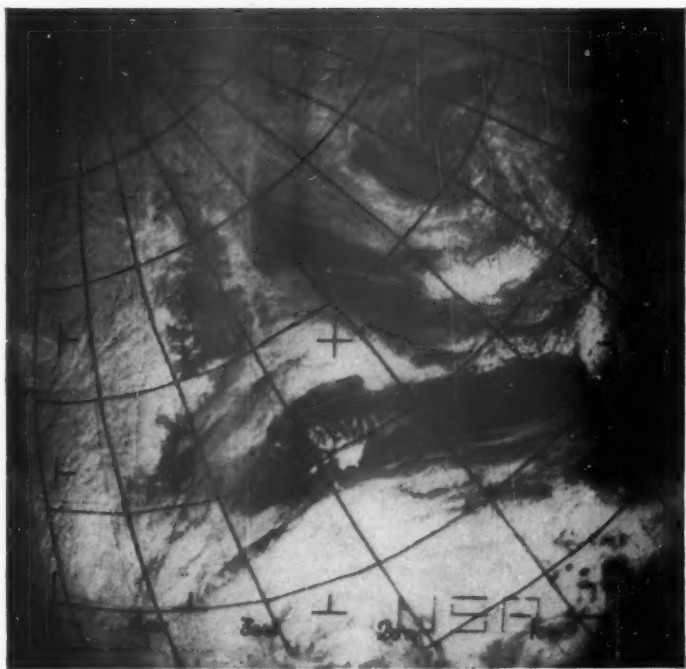


PLATE I—SATELLITE PHOTOGRAPH AT 1230 GMT, 21 SEPTEMBER 1970 SHOWING
EMISSION FROM AN ACTIVE VOLCANO ON JAN MAYEN ISLAND

To face page 37



Photograph by C. S. Broomfield

PLATE II—CUMULONIMBUS MAMMATUS LIT BY THE RAYS OF THE EVENING SUN

AT GRAINAU, BAVARIA, 1840 GMT, 24 JULY 1970

The mountain peak is 1644 metres above sea level.

(a) Cases for which H is independent of θ . In the normal case H is found to be independent of θ over the whole range of values of θ for a given ground track. Note of course that this range does not extend from -180° to $+180^\circ$ but is dependent on M ; e.g. for $M = 1.5$ we find that $-48^\circ < \theta < +48^\circ$, and for $M = 2.0$ then $-60^\circ < \theta < +60^\circ$ (approximately). It can be seen from equation (7) that in these cases V_θ is a continuous function of θ , as shown in Figure 5(a), which is differentiable at all points over the θ range. The

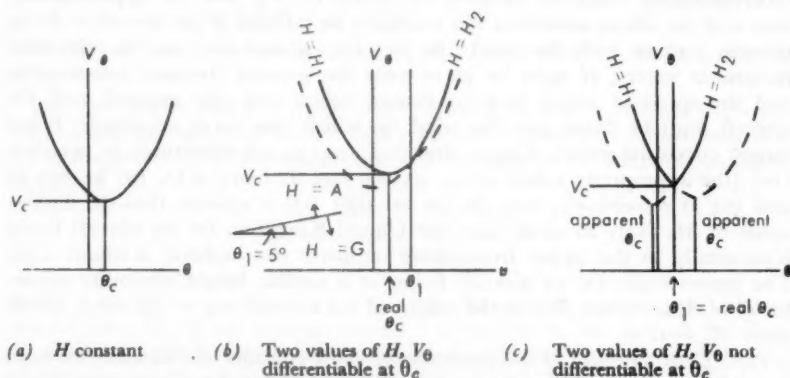


FIGURE 5—RELATIONSHIPS BETWEEN V_θ AND θ

— Real relationships - - - Extrapolated relationships assuming H is constant
 A = Aircraft level G = Ground level

lowest value of V_θ for which the bang reaches the ground (i.e. the airspeed V_c which must be attained for the bang to reach the ground), and the angle θ_c which defines the ray along which the wavefront first does so, can both be found by solving the relationship $\partial V_\theta / \partial \theta = 0$. The solutions are :

$$\sin \theta_c = \frac{v_H - v_A}{a_H} \quad \dots (8)$$

$$\text{and} \quad V_c = a_H + (u_H - u_A) - \frac{(v_H - v_A)^2}{2a_H} - R, \quad \dots (9)$$

where R is a negligible remainder, the maximum value of the ratio R/V_c being 0.0017. Also

$$M_c = \frac{a_H}{a_A} + \frac{(u_H - u_A)}{a_A} - \frac{(v_H - v_A)^2}{2a_A a_H}, \quad \dots (10)$$

With the sign conventions described in Figure 3, u is positive for a tailwind component and negative for a headwind component, and v is negative for winds from the right of the airspeed vector (looking along it) and positive for those from the left. Since M_c is dependent on wind components measured along and normal to the airspeed vector it is obviously dependent on the direction of this vector. If the direction does not change then M_c can be simply and accurately evaluated from equation (10). Usually, however, the

aircraft would be flying on a straight-line ground track and if it is in an atmosphere with a substantial sidewind component, the direction of the airspeed vector will change as it accelerates; to obtain an exact solution of M_e in this case equation (10) would have to be solved simultaneously with an equation describing the aircraft heading.

Since the third term of equation (10) is always negative the highest value of M_e will be found on condition that $v_H - v_A = 0$; if H is at the ground (corresponding variables denoted by suffix G) v_H will be approximately zero and the above condition will normally be satisfied if the aircraft is flying directly into or with the wind. In fact for a_H and $u_H - u_A$ to take their maximum values, H must be at or near the ground (because temperature and the speed of sound have maximum values near the ground) and the aircraft must be flying into the wind (in which case u_A is negative). If the largest coexisting values of a_G/a_A and $(u_G - u_A)/a_A$ are substituted in equation (10) (the appropriate values of u_G , u_A , a_G , and a_A being 6 kt, 197 kt, 644 kt and 564 kt respectively, over the period 1961-67) it appears that the highest value of M_e likely to occur over the United Kingdom, for an aircraft flying horizontally in the upper troposphere or lower stratosphere, is about 1.48. The lowest value for an aircraft flying at a similar height obviously occurs for $H = A$, i.e. when H is at the height of the aircraft ($u_H = u_A$, etc.), which gives $M_e = 1$.

(b) *Cases for which H is dependent on θ .* Now consider the minority of cases for which H is a function of θ , and thus for which the above equations (8-10) do not strictly give valid solutions for θ_e , V_e and M_e . In these cases the form of the relationship between V_θ and θ is as shown in either Figure 5(b) or 5(c) (deduced by changing the value of H in equation (7)), and normally H will take two values as shown, i.e. H_1 for $\theta \leq \theta_1$, and H_2 for $\theta \geq \theta_1$. Since $(a+u)_{H_1} = (a+u)_{H_2}$ for $\theta = \theta_1$, V_θ is a continuous function of θ but it is not a differentiable function at θ_1 . The dotted lines in the two figures are the extrapolated V_θ , θ curves keeping H constant at either value. The 'apparent' values of θ_e (shown against the dotted lines) are still given by equation (8), and by substituting extreme values (i.e. ± 200 kt) for $v_H - v_A$ in the equation it can be shown that both 'real' and 'apparent' values of θ_e lie in the range $-19^\circ \leq \theta_e \leq +19^\circ$. Thus for equations (8), (9) and (10) to give correct solutions it is only necessary for H to be constant for θ in the range $-19^\circ \leq \theta \leq +19^\circ$. In terms of meteorological conditions it is possible for H to vary over this range of values of θ if a tailwind component of 60-100 kt exists at the aircraft (assuming $A > 28\,000$ ft), depending on the temperature structure between the aircraft and the ground.

The case shown in Figure 5(b) is illustrated by the example shown in the inset. For $\theta_1 = 5^\circ$, and $\theta_1 \leq \theta \leq 19^\circ$ height H is at the aircraft and for $-19^\circ \leq \theta \leq \theta_1$ it is at the ground. For rays defined by $\theta > \theta_1$, then $v_H = v_A$ and $\theta_e = 0^\circ$. However, this solution for θ_e does not lie in the range of values of θ ($5^\circ \leq \theta \leq 19^\circ$) for which H is at the aircraft and is thus a non-valid solution. The valid solution for θ_e is obtained by using the value of H for the rays defined by $\theta \leq \theta_1$. As in this example it can be shown that generally the value of H which should be used in these cases is that which pertains to the family of rays which includes that specified by $\theta = 0^\circ$, and the correct value of M_e will be found by substituting the appropriate values of a_H , u_H , and v_H in equation (10).

If neither part of the curve contained a real minimum value as in Figure 5(c), then use of the three equations with either value of H would yield non-valid solutions of θ_c , V_c and M_c ; the valid solution for θ_c is of course $\theta_c = \theta_1$, and θ_1 could be calculated from the atmospheric data. In this case M_c would be given by

$$M_c = \frac{a_H}{a_A \cos \theta_1} + \frac{(u_H - u_A)}{a_A} - \frac{(v_H - v_A) \tan \theta_1}{a_A}, \quad \dots (11)$$

where H can take either of the values H_1 or H_2 , one of which will be pertinent to the $\theta = 0^\circ$ ray.

(c) *Comparison with other work; cases for which $\theta_c = 0^\circ$.* In other literature it has been assumed that θ_c is always zero, and it has been shown here that this is incorrect. The assumption that $\theta_c = 0$ leads to the following equation for M_c :

$$M_c = \frac{a_H}{a_A} + \frac{(u_H - u_A)}{a_A}, \quad \dots (12)$$

which is strictly only valid if $v_H - v_A = 0$. Use of this equation will always overestimate M_c in an atmosphere with sidewinds to the aircraft heading, the maximum possible error being about 0.06; however, for values of $v_H - v_A$ of less than 100 kt the error is < 0.016 .

(ii) *Climbing aircraft.* The previous section considered the Mach number which an aircraft in horizontal flight would have to reach before the bang it was producing locally reached the ground. For a climbing aircraft there exists a cut-off Mach number at each flight altitude, and if the aircraft is flying below that Mach number at any altitude then the bang will not reach the ground from that altitude. Since M_c is wind and temperature dependent it is obviously a function of height; it cannot, however, be simply derived from equations (10) or (12) since the effect of climb angle β must be taken into account. The initial wavenormal geometry for a climbing aircraft is shown in Figure 6, θ now being considered the angle between the x -axis and the projection of the airspeed vector on the x, y plane.

For a climbing aircraft the relationship between l_A and θ is given by

$$l_A = \frac{\cos \theta \cos \beta + \sin \beta (M^2 \cos^2 \theta \cos^2 \beta + M^2 \sin^2 \beta - 1)^{\frac{1}{2}}}{M(\cos^2 \theta \cos^2 \beta + \sin^2 \beta)}, \quad \dots (13)$$

where M is the Mach number of the climbing aircraft. Substitution of this relationship in equation (5) and expansion of the u terms gives

$$\begin{aligned} & \frac{V_\theta (\cos^2 \theta \cos^2 \beta + \sin^2 \beta)}{\cos \theta \cos \beta + \sin \beta (M_\theta^2 \cos^2 \theta \cos^2 \beta + M_\theta^2 \sin^2 \beta - 1)^{\frac{1}{2}}} \\ & = a_H + (u_H - u_A) \cos \theta - (v_H - v_A) \sin \theta, \quad \dots (14) \end{aligned}$$

where M_θ is the Mach number the aircraft must reach before the wavefront reaches the ground along the θ ray.

For small values of β it can be shown, as in the non-climbing case, that θ_c is usually given by equation (8); the proof of this is very arduous and will not be given here, but the result is intuitively obvious from the geometry of

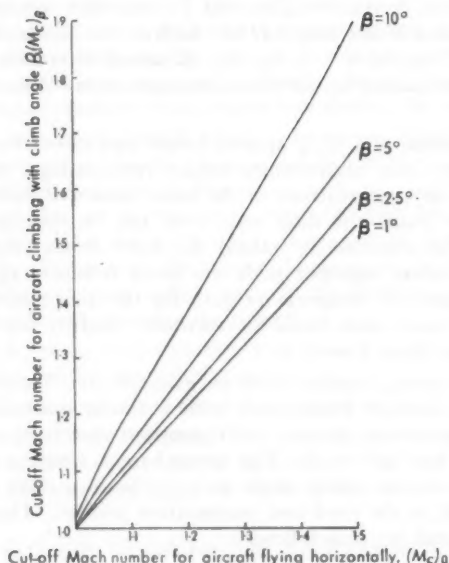


FIGURE 7—VARIATION OF $(M_c)_\beta$ WITH $(M_c)_0$ AND β

$(M_c)_\beta$ for an aircraft climbing at 5° over the United Kingdom are 1.004 and 1.64 for the lower stratosphere or upper troposphere.

A method of calculating $(M_c)_\beta$ by hand. Normally an aircraft accelerating transonically will also be climbing and most of this section is devoted to a climbing aircraft. It should first be emphasized that there would be several advantages in calculating the $(M_c)_\beta$ -height profile by a computer; indeed this would be a necessity if profiles were required for several flights a day especially if acceleration corridors on various headings were used. As well as the advantage of time saving, calculations by computer (which could be based solely on the theory described in this paper) would involve making none of the assumptions and approximations which have to be made in order to derive a useful method of calculation by hand. The method given below is, however, fairly accurate and could be used for non-routine flights.

It is first of all assumed that the aircraft is flying along a straight-line ground track and that it accelerates from $M = 1.0$ to $M = 1.66$ over any height range between 25 000 ft and 45 000 ft. Now $(M_c)_\beta$ is dependent, for a given ground track, on the drift angle α (since α defines the heading, see Figure 3) which in turn is dependent to a slight extent on the airspeed at a given height; however, prescribing the airspeed-height profile would reduce the benefits of calculating the variation of $(M_c)_\beta$ with height since one reason for doing this calculation is to deduce a beneficial airspeed-height or Mach number-height relationship, for the portion of the flight in which the transonic acceleration takes place. In deriving the $(M_c)_\beta$ -height profile it is therefore assumed that the aircraft flies with an airspeed V which corresponds to an

average value of $(M_c)_\beta$ at all heights, and V is therefore assumed to be 700 kt. The final assumption is that height H at which $a + u$ is a maximum is constant over the θ range $-19^\circ \leq \theta \leq +19^\circ$ for all actual directions of the airspeed vector. The errors caused by all these assumptions are discussed in the next section.

The forecast profile of $(M_c)_\beta$ against height can either be based on forecast wind velocity and temperature-height relationships (for the area of the acceleration), or on persistence of the latest measured data. If persistence type forecasts are made the data used need not be simultaneous; greater accuracy would be achieved by taking the latest hourly reports of surface wind and temperature together with the latest 6-hourly upper wind data and 12-hourly upper air temperature data for the area under consideration. The latter temperature data could be (mentally) slightly adjusted to make it fit the other (later) data, if need be.

The method is given, together with an example (see Table I), showing its application to a standard atmosphere with a 160-kt south-westerly wind at 32 000 ft, which decreases linearly (with constant direction) to zero knots at both the ground and 48 000 ft. The ground-track heading is taken in the example as 270° and the climb angle as 2.5° ; both of these quantities must be supplied as well as the wind and temperature profiles. The method, based on equations (6) and (7), is as follows:

- (i) Tabulate the temperature and winds at all significant levels up to 45 000 ft, and at 25 000 ft and 45 000 ft.
- (ii) Estimate v' the wind component normal to the ground track at 25 000 ft, 45 000 ft, and intermediate significant levels (see Table I for application to example). Using Figure 8 calculate the associated aircraft headings and choose an average heading.

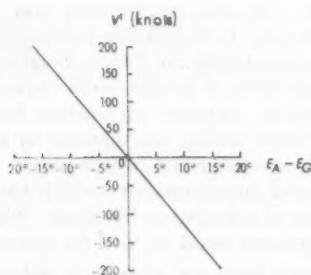


FIGURE 8—CALCULATION OF AIRCRAFT HEADING

E_A Aircraft heading (to be calculated) E_G Ground track heading (given)
 v' Wind component normal to ground track
 Calculated value of E_A is accurate to within $\frac{1}{4}^\circ$ for $\beta \leq 10^\circ$.

- (iii) For this average heading calculate and tabulate the wind components u (along the heading) and v (normal to it) at all tabulated heights. Note the sign convention on Table I.
- (iv) Using tables relating sound speed to temperature, tabulate sound speeds at all the heights and add to the u values to give $a + u$.

- (v) Tabulate the values of H at which the maximum value of $a + u$ occurs between the aircraft and the ground, and tabulate a_H . Note that as the aircraft climbs, with an increasing depth of atmosphere beneath it, H may vary with altitude (but it normally does not). If two equal maxima occur, make calculations for either value of H .
- (vi) For the calculated aircraft heading at 25 000 ft find the wind components along and normal to this heading at the appropriate height H (u_H and v_H) and at 25 000 ft (u_A and v_A). Repeat and tabulate for all heights up to 45 000 ft using the individual headings and appropriate H values each time. Note that if the total change in aircraft heading is $\leq 5^\circ$ the values of u_H , v_H , u_A and v_A can be taken as those (appropriate) values already calculated for the *mean* heading. This considerably shortens the procedure.
- (vii) Calculate $(M_c)_0$ from equation (10), or if $|v_H - v_A| \leq 100$ knots for all heights use the simplified equation (12).
- (viii) Using Figure 7 calculate the $(M_c)_\beta$ -height profile.

The full procedure just described takes about 25 minutes; however, in the large majority of cases step (vi) is unnecessary and use of the simplified equation (12) is adequate, reducing the working-out time to about 15 minutes (after experience).

For flights at a constant aircraft heading, u and v obviously need calculating only for the one heading. For flights at constant altitude the heading may be assumed constant. Figure 9 shows the $(M_c)_\beta$ -height profile calculated for a quarter headwind (as for example in Table I), together with profiles for a direct headwind of 160 kt and a direct tailwind of 160 kt (decreasing to zero knots at the ground and at 48 000 ft); as before, standard atmospheric temperatures were used. The corresponding cut-off airspeeds and groundspeeds are also shown.

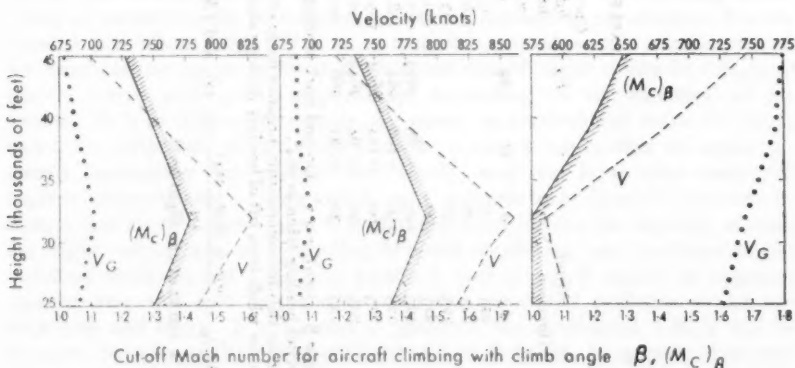


FIGURE 9—CUT-OFF MACH NUMBER, AIRSPEED AND GROUND SPEED
 V , Airspeed V_G Groundspeed

Error analysis. The main sources of error are (i) the provision of incorrect atmospheric data and (ii) the provision of an incorrect climb angle β , or its

TABLE I—CALCULATION OF $(M_c)_\beta$

Height β	Temp. $^{\circ}\text{C}$	Wind Direction	Wind Speed kt	ψ' kt	E_A deg	Components for mean heading		H	Components for individual headings			$(M_c)_0$	$(M_c)_\beta$
						v	u		u_H	u_A	v_H knots	v_A	
β	$^{\circ}\text{C}$	deg	kt	kt	deg	knots	knots						
5 000	+15	225	26	0		0	661						
10 000	+5.8	225	26	0		16	631						
18 000	-2.1	225	50	0		33	641						
25 000	-17.1	225	90	0		59	623						
30 000	-32.9	225	125	89	262.4	82	604	Ground	0	-99	0	+76	1.26
30 000	-44.3	225	150	106	261	98	590	Ground	0	-121	0	+88	1.33
32 000	-46.7	225	160	114	260.4	105	587	Ground	0	-130	0	+93	1.35
39 000	-54.4	225	90	64	264.4	59	577	Ground	0	-69	0	+57	1.27
45 000	-52.5	225	30	21	268	20	579	Ground	0	-22	0	+20	1.18

Notes :
Climb angle β (given) = 2.5° . Ground track heading E_G (given) = 270° .Mean aircraft heading taken as 264° . u Positive for tailwind components, negative for headwind components. v Positive for winds from left of heading (looking along it), negative for those from right.

$$(M_c)_0 = \frac{a_H + (u_H - u_A)}{a_H} \quad (\text{eq. 12}), \text{ and } (M_c)_\beta = \frac{a_H + (u_H - u_A)}{a_A} - \frac{(v_H - v_A)^2}{2a_A a_H} \quad (\text{eq. 10}) \text{ for all calculations when}$$

 $v_H - v_A > 100 \text{ kt at any flight level.}$

variation with height. These sources will of course cause error no matter what method of calculation is used. Even if correct data are provided the particular method described above may introduce errors due to (a) differences in aircraft heading from those used (assuming an airspeed of 700 kt), (b) the existence of two values of H over the θ range and (c) the use of the shortened formula. These three sources are the least common and will be dealt with first.

Errors due to the use of incorrect headings or drift angles (which are dependent on the airspeed for a given ground track) are unavoidable as already explained; for this reason $(M_e)_\beta$ can be underestimated by a maximum of 0.03 at the height for which the actual Mach number is unity and, at the most, overestimated by the same amount for the height at which it is 1.5; these errors are slightly dependent on β also but it is assumed in this section that $\beta \leq 2.5^\circ$.

The existence of two values of H , and the use of one of them to deduce $(M_e)_\beta$ (in the case depicted by Figure 5(c)) can result in an underestimate by a maximum of 0.07, or a maximum overestimate of 0.03; these errors are inclusive of any due to simultaneous use of an incorrect heading. In fact this source of error will seldom occur, especially for westbound flights when a 60-90-knot easterly wind component at the aircraft would normally be required to produce two similar H values (i.e. the aircraft height and the ground). Even then the wind speed at the aircraft would need to be about 200 kt to produce the errors referred to. It is only necessary, therefore, to remember this source as a possible but infrequent reason for large error.

The maximum error due to the use of the shortened formula is about 0.02. The average value of the errors resulting from the three sources discussed above is <0.002 in each case.

The most frequent source of error in a calculated value of $(M_e)_\beta$ would be the use of incorrect atmospheric data resulting from a forecasting error; errors so caused would be independent of the method of calculation. For the normal case where height H is at or near the ground, vector wind errors of, for example, 60 kt or 20 kt at the aircraft would cause errors in $(M_e)_\beta$ of up to ± 0.10 and ± 0.03 respectively, depending on the direction of the vector. It is of interest to deduce the errors in a calculated value of $(M_e)_\beta$ based on persistence of measured data for a time 6 hours after an upper air ascent (measuring temperature and wind), and one hour after measured surface observations. Using wind and temperature data^{4,5,6} it can be shown that for the cases where H is at the ground (by far the majority of cases for flights which are on a heading between south-west and north-west) the standard deviations of $(M_e)_\beta$ at 30 000 ft and 40 000 ft would be approximately 0.03 and 0.02 respectively, and the associated probable errors are thus 0.02 and 0.013. If the errors at 30 000 ft are considered, $(M_e)_\beta$ will be in error, because of the use of persistence type forecasts, by greater than 0.08 on one occasion in 100 and by greater than 0.04 on one occasion in 20. The probable errors calculated above will also be the approximate values for all cases, since a large majority of cases fall into the family for which the calculations were made.

Errors resulting from the use of an incorrect value of β are a function of both β and (M_e) , but they can be deduced directly from Figure 7 and may be

additive to the errors resulting from the use of incorrect atmospheric data. For example for $\beta = 2.5^\circ$, $(M_c) = 1.5$, an error of 1° in β will cause an error of ± 0.03 in $(M_c)_\beta$.

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551-553-21

THE STRANGE WINDS OF RAS ASIR (FORMERLY CAPE GUARDAFUI)

By J. FINDLATER

Summary. The complex wind structure near Ras Asir (formerly Cape Guardafui) during the northern summer has been examined using surface and upper-wind reports from adjacent stations and sea areas. It is deduced that the pronounced horizontal and vertical variation of wind is consistent with a persistent and mesoscale eddy linked to the environs of Ras Asir, and embedded in the edge of the strong flow of the south-west monsoon.

Introduction. A recent study of the airflow at low levels over eastern Africa and the western Indian Ocean (Findlater 1971) analysed a low-level air current which lies near the western boundary of the (northern) summer monsoon circulation. The core of the current can first be identified to the east of Madagascar and from there the current enters the northern hemisphere over the low-lying eastern areas of Kenya. It passes through eastern Ethiopia and Somalia before leaving the continent of Africa near Ras Hafun to cross the Arabian Sea and reach India. The mean speed of the core is about 15 m/s at 1500 m but speeds of between 25 and 50 m/s have been recorded occasionally.

More recently all available pilot-balloon data in the area have been examined to derive monthly mean winds for levels between 1000 m and 3000 m and define the climatological framework within which the high-speed current develops. The results are to be published in the near future (Findlater²), but Figure 1 reproduces a small and simplified section of this analysis in the vicinity of Ras Asir (formerly known as Cape Guardafui) for the 1000-m level in July — neglecting mesoscale distortions. The names of some stations are also included in Figure 1(a).

Ras Asir — the strange winds. A pilot-balloon station operated from January 1930 to June 1933 near the lighthouse at Ras Asir. The balloon ascents were made from the settlement at Tohèn, position $11^\circ 44' N$, $51^\circ 15' E$

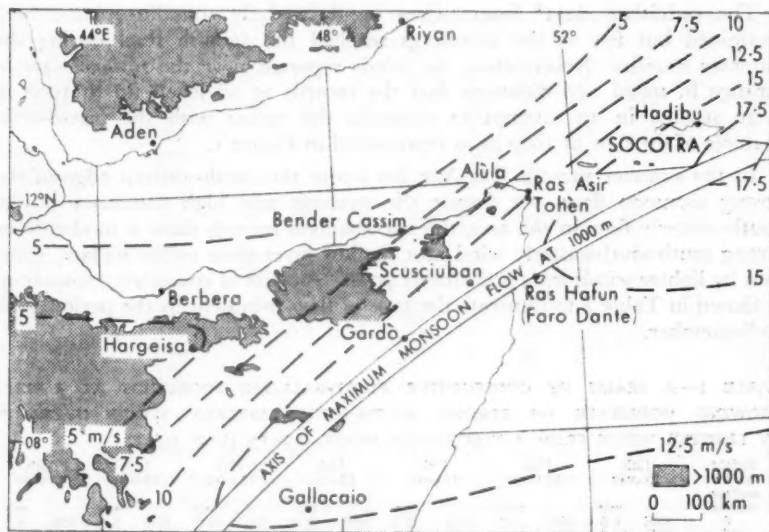


FIGURE 1(a)—MEAN FLOW AT 1000 m IN JULY, AND LOCATION OF STATIONS

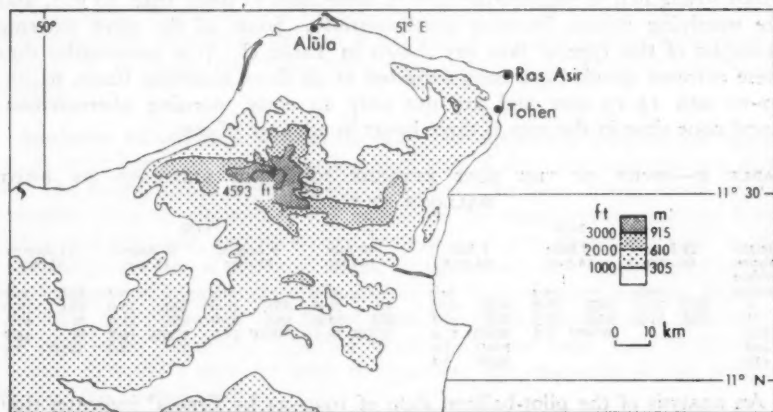


FIGURE 1(b)—TOPOGRAPHY OF AREA

and height 80 m above MSL. The lighthouse is 11 km to the north of Tohen with its base 244 m above MSL at $11^{\circ} 49' 30'' \text{N}$, $51^{\circ} 16' 10'' \text{E}$. The positions of the two stations are included in Figure 1(b) which shows the detailed topography of the area. Of particular interest is the arc of high ground which lies aslant the summer monsoon flow.

The published data⁴ from twice- or thrice-daily soundings have been examined but few of the soundings reached the 1000-m level during the summer months. Nevertheless, the winds reported near the surface were so strange in speed and direction that the records at all levels up to 1000 m were studied in an attempt to reconcile the values with the broad-scale features of the flow at 1000 m as represented in Figure 1.

In the summer months Ras Asir lies under the north-western edge of the strong monsoon flow, yet despite the strength and high constancy of the south-westerly flow in the area the pilot-balloon records show a moderate or strong south-south-easterly wind in a shallow layer close to the surface, overlain by lighter winds from a northerly point. A series of consecutive soundings is shown in Table I to illustrate the type of flow prevalent in the period June to September.

TABLE I—A SERIES OF CONSECUTIVE PILOT-BALLOON SOUNDINGS AT TOHÈN, SHOWING MODERATE OR STRONG SOUTH-SOUTH-EASTERLY WINDS OVERLAIN BY LIGHTER WINDS FROM A NORTHERLY POINT, 12–15 JULY 1932

Height above surface metres	12th 15 GMT	13th 10 GMT	13th 15 GMT	14th 10 GMT	14th 15 GMT	15th 10 GMT	15th 15 GMT
0	SSE 8.0	SSE 8.5	SSE 8.5	SSE 28.0	SE 10.0	SSE 5.5	SSE 8.5
75	SSE 14.8	SSE 13.8	SSE 10.8	SSE 28.2	SSE 11.5	SSE 12.8	SSE 17.5
150	SE 9.6	SSE 9.2	SSE 11.3	NW 13.6	SSE 6.5	SSE 10.0	SSE 12.6
300	N 3.9	SE 6.5	NE 1.2	NNW 4.8	N 0.7	NE 2.6	N 1.0
450	ENE 6.0	NNW 11.8	NNW 2.2	NNW 4.6	N 2.0	NE 3.3	NW 3.6

On many occasions, however, the shallow low-level south-south-easterly winds strengthen to remarkable speeds, sometimes to more than 50 m/s, and the overlying stream becomes north-westerly. Some of the most extreme examples of this type of flow are shown in Table II. It is noteworthy that these extreme speeds have been recorded at all three sounding times, 04–05, 09–10 and 14–15 GMT and are not only an early morning phenomenon. Local zone time in the area is three hours in advance of GMT.

TABLE II—SEVEN OF THE MOST EXTREME SOUNDINGS RECORDED BY PILOT BALLOON AT TOHÈN

Height above surface metres	1930			1932				
	29 June 04 GMT	30 June 14 GMT	1 July 04 GMT	16 July 10 GMT	9 August 15 GMT	10 August 15 GMT	14 August 10 GMT	
0	SSE 21.0	SSE 21.0	SSE 25.3	SE 20.0	SSE 16.0	SSE 18.0	SSE 18.0	
75	SSE 51.0	SSE 37.5	SSE 37.5	SSE 54.0	SSE 42.0	SSE 51.0	N 47.6	
150		WNW 5.0	NNW 11.4	NNW 38.4	NNW 21.0	NNW 16.7	W 16.9	
300			NW 5.6			E 1.3	WNW 2.4	
450			SSW 3.1					

An analysis of the pilot-balloon data of 1930–33 by Eredia⁵ indicates that in July the most frequent wind directions at specified heights above the station were :

Height (m)	0	75	150	300	450	600	750	900
Direction	SSE	SSE	SSE	Var.	Var.	WNW	WNW	WNW

Eredia comments that south-westerlies were almost non-existent at the surface in the summer months of 1930–33 at Tohèn, but Fantoli⁶ has published summaries of surface wind directions at the lighthouse for the combined periods 1936–39 and 1954–58 which show that south-westerlies occurred with almost the same frequency as south-easterlies, whilst southerly winds

TABLE III—SURFACE WIND DIRECTION FREQUENCIES DURING JUNE, JULY AND AUGUST

Station and period	Month	N	NE	E	SE	S	SW	W	NW	Calm	Total
Scusciuban 1953-58	June	1	42		2	30	198	261	108	10	652
	July					10	391	261	71		740
	Aug.	1			5	54	410	205	34	7	716
Gardò 1938 1953-58	June					248	490	12			750
	July					247	285	243			775
	Aug.					261	464	19			744
Bender Cassim 1934-39 1953-58	June	62	322	11	89	231	230	5	128	62	1140
	July	96	163	4	67	291	206	18	147	104	1096
	Aug.	109	125	2	43	251	175	25	195	90	1015
Alùla 1953-58	June	159	156	6	1	1		3	23	223	572
	July	112	184	95	7	25	5	21	12	147	608
	Aug.	126	265	33	15	11	13	20	16	121	620
Ras Asir 1936-39 1954-58	June	6		5	243	136	249	5	9	157	810
	July				321	91	337			14	763
	Aug.		1	3	373	141	302	3	1	28	852
Faro Dante 1936-39	June				20	70	90	90			270
	July						279				279
	Aug.						279				279

were of much lower frequency (see Table III, Ras Asir). Because of the differing positions and altitudes of the two stations, the lack of site details and the difference in periods, it is difficult to explain the discrepancy between the two sets of data. One plausible explanation would be the existence of persistent small-scale eddies tied to the local topography, and another would associate the south-westerlies at Ras Asir, which is 164 m higher than Tohèn, with the layer of changing direction evident at about 150 m above Tohèn when winds are strong, as indicated in Table II. However, despite the lack of an adequate explanation of the south-westerly periods at Ras Asir and their non-occurrence at Tohèn, the fact remains that at both stations south-south-easterly or south-easterly flow, often very strong, is of frequent occurrence in July.

Analysis of surface winds. It might be suspected, because the upper winds were based on an assumed rate of rise of the pilot balloons, 2.5 m/s, that the strong winds were unreal. However, the very strong south-south-easterlies close to the surface are confirmed by anemometer readings from Tohèn and Ras Asir, and by reports from ships of very strong southerly winds near and to the east of the cape, where south-westerly flow might be expected. The long-period mean surface winds over the Gulf of Aden and the Arabian Sea from the atlases of the Royal Netherlands Meteorological Institute^{7,8} and the most frequently reported wind directions from land stations in Somalia (Fantoli⁹) may be used to represent the character of the surface flow in July, as in Figure 2.

The short arrows over the land show the most frequently reported wind directions over land, based on three observations daily, and those over the sea the mean direction and mean speed in metres per second (converted from data published in Beaufort forces) within the circle denoting the middle of the sea area to which each report refers. These areas are 2-degree quadrangles eastward from 50°E and 1-degree quadrangles westward into the Gulf of Aden.

The striking feature of Figure 2 is the predominant north-easterly wind at Alùla, confirming that flow near Ras Asir is complex. The observations from

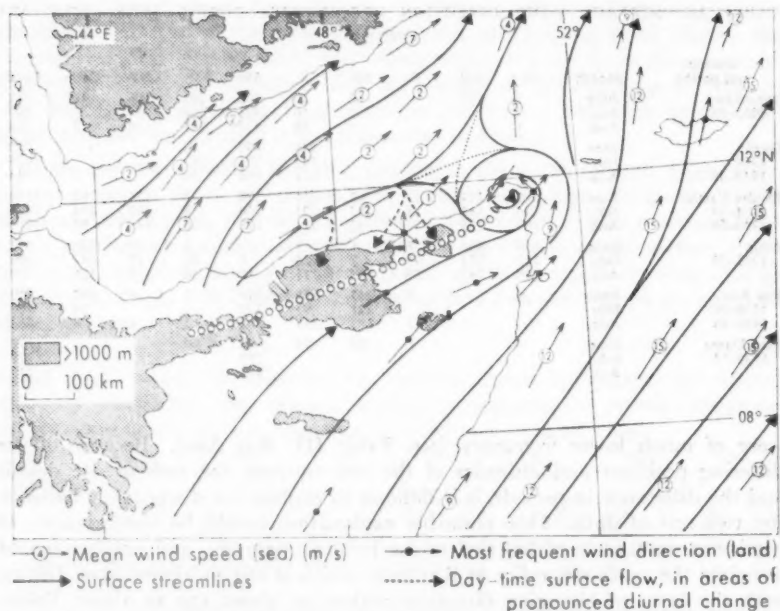


FIGURE 2—MEAN SURFACE FLOW IN JULY

oooooo Sea-breeze convergence zone

which the most frequent wind direction at Alùla was calculated by Fantoli⁶ were made at 04, 10 and 15 GMT, yet the most frequent direction was from the north-east and the least frequent from the south-west.

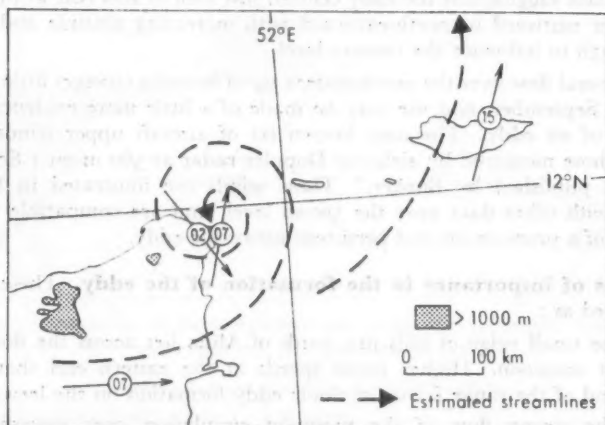
Surface wind data for Alùla and nearby stations, also extracted from Fantoli's work, are shown in Table III for the months of June, July and August. It can be deduced that the small number of reports of south-westerlies at Alùla in all three months preclude the existence of a significant number of south-westerlies at 04 GMT. The prevailing north-easterlies are therefore not due to a simple sea-breeze system. For example, Table III includes 186 observations at 04 GMT (dawn) in July and most of these winds come from a northerly point.

Figure 2, analysed by streamlines, reveals a small-scale eddy centred between Ras Asir and Alùla, on the north-western edge of the main monsoon stream. Though the diameter of the closed circulation is only about 200 km its influence is seen in the deformation of the flow over a much wider area of about 500 km diameter. Winds at Scusiuban are veered and those near Socotra are backed from the broad-scale flow. Also, an area of light mean winds lies to the north-west side of the eddy.

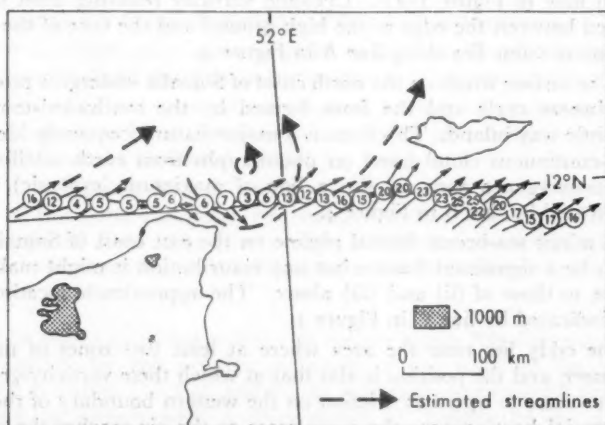
Analysis of upper winds. No attempt is made here to challenge the validity of the winds measured by pilot balloons from Tohèn. Any attempt to do so would probably be inconclusive and enable the strange effects to be

dismissed without adequate explanation. Rather, the approach in this paper is to demonstrate that the observed values, especially those of direction, are quite consistent with the mesoscale eddy noticed in the monthly mean surface wind field.

At the 450-m level the mean wind in July 1930 was 190° , 6.6 m/s and in July 1932 was 325° , 2 m/s and both of these values are plotted in Figure 3. In July 1931 no soundings were followed beyond the second minute of the ascent, 300 m. These values compared with a July mean for Socotra at 500 m



(a) Mean winds at 450 or 560 m in July (June at Scusiuban); wind speed (m/s) in centre of circle, wind direction shown by short arrow.



(b) Doppler radar winds at 560 m on 1 September 1964 (after Bunker³).

The aircraft position is marked by a circle with wind speed (m/s) in the centre and a short arrow to show wind direction.

FIGURE 3—SOME FRAGMENTARY WIND DATA AT 450–560 m NEAR RAS ASIR DURING THE SUMMER MONTHS

of 200° , 15 m/s for the years 1942-45, and the well-known strong south-westerly winds south and east of Ras Asir, indicate the existence of a lighter and variable wind area or eddy at 450-500 m in the area. Furthermore, the July mean wind at 1000 m at Tohèn is 280° , 5 m/s and between June and September varies only in the range $270-285^\circ$, 4.5-5 m/s, though there are barely sufficient data at this level to determine a true mean value. Nevertheless, westerly to north-westerly winds must exist on a considerable number of occasions.

These data suggest that the eddy centred just west of Ras Asir at the surface lies further eastward or north-eastward with increasing altitude and extends high enough to influence the 1000-m level.

The general flow over the north-eastern tip of Somalia changes little between June and September, and use may be made of a little more evidence for the existence of an eddy. The only known set of aircraft upper winds at Ras Asir are those measured by airborne Doppler radar at 560 m on 1 September 1964 and published by Bunker.⁹ These winds are illustrated in Figure 3 together with other data near the 500-m level, and are compatible with the existence of a pronounced and persistent mesoscale eddy.

Factors of importance in the formation of the eddy. These may be summarized as :

(i) The small ridge of hills just south of Alùla lies across the flow of the south-west monsoon. Higher mean speeds at the eastern end than at the western end of the range favour cyclonic eddy formation on the leeward side.

(ii) The strong flow of the monsoon circulation over eastern Africa, especially Kenya and eastern Ethiopia, brushes against the high mountains and plateaux and generates large horizontal shears by lateral friction (see July mean flow in Figure 1(a)). Cyclonic vorticity resulting from the shear is contained between the edge of the high ground and the core of the current. The maximum value lies along line A in Figure 4.

(iii) The surface winds on the north coast of Somalia undergo a pronounced daily sea-breeze cycle and the front formed by the southward-moving sea air lies a little way inland. This front is a major feature frequently identifiable as a semi-continuous cloud band on photographs from earth satellites. The front, or convergence zone, is also a line of maximum (cyclonic) vorticity and is indicated by line B in Figure 4.

(iv) A minor sea-breeze frontal régime on the east coast of Somalia is not thought to be a significant feature but any contribution it might make would be additive to those of (ii) and (iii) above. The approximate location of this system is indicated by line C in Figure 4.

(v) The eddy lies near the area where at least two zones of maximum vorticity meet, and the position is also that at which these vorticity-generating mechanisms cease to operate; friction on the western boundary of the stream and differential heating near the coast cease as the air reaches the coast. It is probable that the eddy forms there to exhaust the cyclonic vorticity continuously advected into the area from the south-west quadrant.

The structure of the eddy. Some features of the eddy which can be deduced or inferred from surface and upper wind data are :

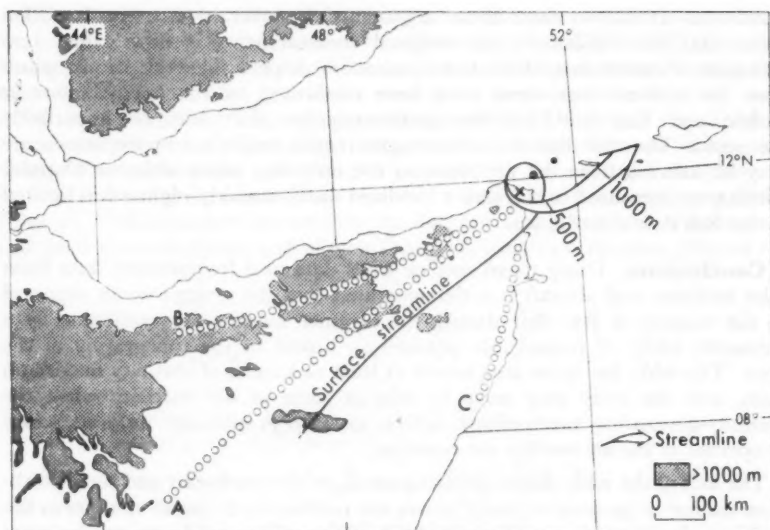


FIGURE 4—SCHEMATIC REPRESENTATION OF ZONES OF MAXIMUM VORTICITY IN RELATION TO THE MESOSCALE EDDY

ooooo Zones of maximum (cyclonic) vorticity

Approximate position of eddy centre : x at surface
o at 500 m
• at 1000 m.

- (i) It is linked to the topography of Ras Asir and environs.
- (ii) It is evident only in the summer months and is sufficiently persistent between June and September to show up in monthly mean values.
- (iii) The eddy is of mesoscale dimensions, having a diameter at the surface of about 200 km though it is associated with distortion of the wind field over an area of about 500 km diameter.
- (iv) It extends upward to between 500 and 1000 m and the axis slopes to the north-east with increasing altitude. The slope of the axis, deduced from fragmentary data, is about 1:100 although there is no direct evidence that the flow is a closed circulation at the upper levels.

An eddy with a quasi-horizontal axis sloping across Ras Asir would provide an explanation for the peculiar structure of the wind in the vertical as illustrated in Tables I and II, and for the previously unexplained simultaneous occurrence of surface south-south-easterlies at Tohèn (80 m) and south-westerlies at Ras Asir (244 m). However, the eddy is likely to generate upward motion near its centre to connect the flow at various levels and the present evidence suggests that the three-dimensional flow, illustrated schematically in Figure 4, may be similar to that experienced, i.e. a very strong south-south-easterly or southerly overlain by a lighter wind from the north-west

quadrant. It follows from these arguments, however, that vertical motion exists and the validity of the original measurements is now called into question. Nevertheless, there is no reason to suspect the very strong winds near the surface since these have been confirmed by the anemometers at Tohèn and Ras Asir, but the north-westerlies aloft may be imperfectly measured. That the direction of the upper régime might not be too inaccurate may be inferred from the fact that on the only day when airborne Doppler winds were measured in the area a localized north-westerly régime was located in the Ras Asir disturbance.

Conclusions. Using mean surface wind data and fragmentary data from pilot balloons and aircraft it is demonstrated that the strange winds reported in the vicinity of Ras Asir during the summer months are explicable by a mesoscale eddy, of remarkable persistence, linked to the topography of the area. The eddy lies in an area where at least two zones of vorticity maximum meet, and the eddy may serve to exhaust some of the vorticity when the vorticity-generating mechanisms, which are topographically induced, cease to operate as the air reaches the coastline.

The axis of the eddy slopes gently upwards to the north-east and the disturbance cannot at present be traced above the 1000-m level. Small changes in the position, size, intensity, and angle of tilt of the eddy would account for most of the strange effects which have been reported, and their day-to-day changes.

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551-509-323;551-524-36

FORECASTING MAXIMUM AND MINIMUM TEMPERATURES OVER THREE DAYS

By S. G. ABBOTT

Summary. The practical approach to a requirement for forecasts of maximum and minimum temperatures for each of three consecutive days ahead over a large area in south-west England is discussed and the forecast errors analysed. A regression equation technique based on climatological mean temperatures is shown to give some improvement in forecasts for the third day, with a worthwhile reduction in the number of large errors.

The requirement. Temperature forecasts for sets of three consecutive days were supplied to the South West Electricity Board (SWEB), Taunton Group, by the Meteorological Office at Plymouth/Mount Batten during the winter of 1968/69. The opportunity was taken to prepare the forecasts over a full year to test the forecasting techniques used. The stated requirement was the supply by 09 clock-time daily of area average forecasts in degrees Celsius of maximum and minimum temperatures for the next 24 hours (Day 1) and the following 24 hours (Day 2), with trends for a third 24-hour period (Day 3). The total area covered by the forecasts was about half of Somerset, and parts of north Devon and of west Dorset, as shown on the map (Figure 1).



FIGURE 1—MAP SHOWING SOUTH WEST ELECTRICITY BOARD AREA

The forecasting problem. Over such an area large variations of maximum and minimum temperatures can occur from place to place. The only available regularly reporting stations were Exeter Airport, Morecombelake, Yeovil and Chivenor. Exeter and Yeovil are probably reasonably representative of the main inland centres of population, with Chivenor representing the northern coastal zone and Morecombelake the Lyme Bay coastal zone. It was therefore thought reasonable to reduce the problem to providing a mean maximum and minimum temperature forecast for these four stations for each 24-hour period. Forecasts for the first period were prepared using current synoptic data and forecast charts (prepared by Central Forecasting Office (CFO), Bracknell) for midnight. Day maximum temperatures were forecast using the Gold-square techniques,¹ as modified by Jefferson^{2,3} and Johnston,⁴ on suitable days. Night minimum temperature forecast techniques used were those developed by Tinney and Menmuir,⁵ based on the work of Saunders^{6,7} and Barthram,⁸ which have been found to give good results in this area. The forecasting problem for Days 2 and 3 is similar to that facing CFO when providing weekend guidance forecasts for the Central Electricity Generating Board (CEGB). The 48- and 72-hour forecast charts for midnight, issued by CFO on the previous afternoon, were

used to assess air-mass changes and parameters for use with the techniques employed for Day 1, taking into account any major changes in the forecasts suggested by the latest current synoptic data and forecast charts.

Analysis of forecast errors. An analysis of similar forecasts issued for part of the previous winter had indicated that subjective forecasts were consistently better than persistence forecasts. The present analysis was therefore performed in two parts :

- (i) The forecast maximum and minimum temperatures were compared with the area mean values, calculated from the daily maximum and minimum temperatures recorded at the four stations.
- (ii) To check a technique currently in use in the Meteorological Office for forecasting temperatures for CEEGB for the third day of week-end periods, revised forecasts for Day 3 were prepared on the basis of the following regression equation :

$$\text{Day 3C forecast} = \frac{\text{Day 2 forecast} + \text{climatological mean}}{2}$$

The climatological mean was obtained from the mean maximum and minimum temperatures at Exmouth, Cullompton and Ilfracombe for 10-day periods.⁹

Discussion of results. The results of the analysis are shown in Tables I and II along with the errors obtained when the regression equations are used to produce the forecast values listed as Day 3C. The mean errors indicated that there was no significant tendency to bias forecasts except in January, when the forecast temperatures were biased on the low side for Days 2 and 3. The mean deviations, indicating the mean magnitude of errors irrespective of sign, and the root-mean-square errors, indicating the spread of errors, increased from Day 1 to Day 3 except in September. The extreme error ranges showed little systematic variation apart from the expected increase beyond Day 1. Occasional large errors occurred on all three days, reflecting the synoptic difficulties. Errors in forecasting the tracks and development of depressions from the Atlantic were the main source of large errors. The very large ones in December arose from a 2-day period during which a very mild warm sector occluded over the South-west Approaches and south-west England eventually had cold easterly winds. The revised forecasts for Day 3 (Day 3C) showed decreases in mean deviations and root-mean-square errors but large errors were not eliminated entirely.

Applying the *t*-test to the two sets of forecast errors for Day 3 indicated that there were significant differences at the 5 per cent level in only two of the months. When considering errors of 5 degC or more, however, the regression-equation technique reduced the total of such errors in Day 3 forecasts by about two-thirds and to this extent was a worthwhile improvement. It is interesting to compare the mean errors and root-mean-square errors for the minimum temperature forecasts with those obtained during a test of forecasting techniques in eastern England for the period October 1963 to March 1964, described by Gordon and Virgo.¹⁰ The forecasts for Days 1, 2 and 3 bear comparison with these, considering that the eastern England forecasts

TABLE II.—ERRORS IN FORECAST MINIMUM TEMPERATURES (AREA MEAN MINUS FORECAST TEMPERATURE)

Month	Mean errors			Mean deviation			Root-mean-square errors			Extreme error range		
	Day 1	Day 2	Day 3	Day 1	Day 2	Day 3	Day 1	Day 2	Day 3	Day 1	Day 2	Day 3
1968												
Sept.	+0.3	+0.0	+0.5	-0.2	1.4	1.5	1.8	1.5	1.7	3 to -3	3 to -3	5 to -4
Oct.	+0.1	+0.1	+0.6	+0.9	1.2	1.3	1.6	1.6	1.7	4 to -3	3 to -3	5 to -4
Nov.	-0.3	-0.1	+0.2	-0.6	1.7	1.9	3.1	2.0	2.7	2 to -4	7 to -6	7 to -8
Dec.	+0.2	+0.7	-0.5	-1.3	2.2	2.9	3.6	2.5	2.8	5 to -6	5 to -13	8 to -11
1969												
Jan.	+0.6	+1.1	+2.1	+1.5	2.0	2.0	2.9	2.0	2.5	6 to -4	6 to -4	7 to -3
Feb.	+0.7	+0.6	+0.7	-1.2	1.1	1.9	2.8	2.1	1.2	3 to -1	5 to -4	6 to -5
Mar.	+0.1	-0.1	-0.2	-0.9	1.7	2.1	2.3	1.7	2.0	4 to -3	5 to -4	5 to -6
Apr.	+0.8	+0.5	-0.2	0.0	1.6	2.6	2.9	2.1	1.9	5 to -3	5 to -5	5 to -7
May	-0.4	-0.4	-0.3	-0.3	1.2	1.5	2.0	1.3	1.8	3 to -7	4 to -6	4 to -6
June	-0.2	-0.1	+0.2	-0.6	1.6	1.8	2.3	1.5	2.0	6 to -3	6 to -6	5 to -8
July	+0.3	+0.2	+0.1	0.0	1.6	1.7	1.7	1.7	1.4	3 to -2	4 to -4	4 to -4
Aug.	+0.1	+0.1	-0.7	-0.5	1.5	2.2	2.3	2.0	1.9	5 to -5	6 to -4	6 to -6

Day 3 Day 3C

58 18

Total errors of 5 degC or more

were made for single nights at times more synoptically convenient, when some of the parameters had been recorded. The occasional large errors in the forecasts for Days 2 and 3 could have been serious for Electricity Authority load estimations but the expected advances in synoptic forecasting should eventually reduce such errors.

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AWARDS

L. G. Groves Memorial Prizes and Awards

The 24th award of the prizes was made on Friday, 27 November 1970, at the Ministry of Defence, Whitehall, by Major and Mrs K. G. Groves; the award is in memory of their son, Sergeant Louis Grimble Groves, RAFVR, who was killed on a meteorological sortie in 1945. The Vice-Chief of the Air Staff, Air Marshal Sir Denis Smallwood, presided at the ceremony. (See Plates III, IV and V.)

The Aircraft Safety Prize has been awarded jointly to Squadron Leader A. P. Fletcher and Squadron Leader D. J. Phillips, formerly of Royal Air Force Laarbruch, with the following citation:

'There have been many instances of birdstrikes on the nose of Canberra aircraft and in some cases the shattering of the nose transparency has caused injury to the navigator in the nose prone position. Squadron Leader Fletcher and Squadron Leader Phillips devoted much thought and energy to a means of alleviating this situation and conceived the idea of a transparent screen between the nose transparency and the navigator's face. The 'Macrolon Screen' which they created is now fitted in all Canberra B(1)8s of RAF Germany and not only achieves the primary object of shielding the navigator but also affords a measure of protection to the aircraft. It represents the first worthwhile advance for some time in the sphere of birdstrikes.

By diligent application to a serious problem Squadron Leader Fletcher and Squadron Leader Phillips have made a very valuable and practical contribution to flight safety.'

The Meteorology Prize has been awarded jointly to Mr R. A. S. Ratcliffe, Senior Principal Scientific officer, and Mr R. Murray, Principal Scientific Officer, Meteorological Office; the citation reads :

'In recognition of their work in determining the relationships between anomalies of sea temperature in the Atlantic and subsequent long-term anomalies in the large-scale atmospheric circulation. Relationships between the temperature of the Atlantic and subsequent weather have been suspected for many years but Messrs Ratcliffe and Murray have for the first time demonstrated conclusively the existence and nature of such relations. They have also shown how use can be made of them for the improvement of prediction of weather anomalies over the British Isles for periods of a month or more ahead.'

The Meteorological Observers' Award has been awarded to Mr D. P. Smith, Senior Experimental Officer, Meteorological Office, the citation reading :

'The Board of Trade has for the past two winters maintained a weather advisory ship, the *Orsino*, north of Iceland, to keep contact with British trawlers in that area and to provide them with weather forecasts and warnings. Mr D. P. Smith served as meteorologist aboard the *Orsino* for over 4 months during the 1968-69 winter and did the first tour of duty in 1969-70.

His zeal and devotion to duty contributed largely to the success of the operations, and his pioneering activity played a notable part in the development of satisfactory procedures under working conditions that were often very difficult. The activities of the *Orsino* undoubtedly increased the safety of British trawlers off Iceland and the weather advice provided by Mr Smith played a significant part in making the operations successful.'

The second Memorial Award has been awarded to Mr D. N. Axford, Principal Scientific Officer, Meteorological Office, with the following citation :

'Mr D. N. Axford has made important contributions to the understanding of small-scale air motions in the free atmosphere. Under his direction the complex measuring capabilities of the Canberra aircraft of the Meteorological Research Flight have been developed into a system of remarkable precision for determining the motion of the air in three dimensions. The observations which have thus been obtained with an accuracy not previously achieved have given new insight into the air motions which contribute to the elusive but important phenomena of clear-air turbulence.'

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OBITUARY

Professor Richard Scherhag

Professor Richard Scherhag died on the last day of August 1970 after only a short illness. He was then almost 63 years of age and had been as active as ever in his remarkably energetic and productive work as a Professor in the Free University of Berlin and Director of the Institute for Meteorology and Geophysics, an appointment which he had taken up in 1951. His work in

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PLATE III—AWARD WINNERS WITH MAJOR AND MRS K. G. GROVES AND AIR MARSHAL SIR DENIS SMALLWOOD

Left to right: Squadron Leader A. P. Fletcher, Squadron Leader D. J. Phillips, Mr D. N. Axford, Major and Mrs K. G. Groves and Air Marshal Sir Denis Smallwood (see page 59).

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PLATE IV—MAJOR K. G. GROVES WITH MR R. A. S. RATCLIFFE (CENTRE) AND MR R. MURRAY (LEFT) JOINT WINNERS OF THE MEMORIAL PRIZE FOR METEOROLOGY
(See page 59.)



PLATE V—MAJOR K. G. GROVES WITH MR D. N. AXFORD, WINNER OF THE SECOND MEMORIAL AWARD
(See page 59.)

Berlin was quite outstanding and indeed unique, for his institute was responsible not only for a heavy programme of teaching and research as a university department but also for a weather service for Berlin, complete with synoptic weather communications, chart construction analysis and a public forecasting service. He had of course his own observation station, very well equipped, including a radiosonde station which was perhaps the first in the world to become completely computerized. His satellite cloud mosaics produced most efficiently from the output of U.S. satellites were also a unique and impressive technical accomplishment. They were circulated to subscribers to his 'daily meteorological bulletin' and 'upper air charts', a set of publications, known and valued throughout the world, which covered the whole northern hemisphere and which was extended in recent years to 30 mb and 10 mb. That so much could be accomplished by an institution working with the handicaps peculiar to the Berlin of our day continued to astonish all who were aware of his position. That in his last years he should not have escaped some of the extra pressures brought about by widespread student unrest was indeed an injustice for one who had given so much. His regard for his students was one of his endearing qualities and over some years he would use weeks of his vacation in visiting other countries (Britain amongst them) with a group of students.

Professor Scherhag was of course ideally qualified for the combination of academic work and public service which occupied the last half of his professional life for previously he had, in peace and war, much varied experience starting in 1933 at the famous Deutsche Seewarte in Hamburg. Already before the 1939 war he had attracted international notice by many papers in the German literature but it was when exchange of scientific work became established again after the war that Scherhag's name became known everywhere. First perhaps he will be thought of as a synoptic-dynamical meteorologist, and anyone who had the privilege of seeing Scherhag in action with the analysis of current synoptic charts has witnessed that scientific art at its best. But his published work must be his permanent monument and it is a notable edifice, with some 150 entries in the library of the Meteorological Office. His textbook 'New methods of weather analysis and forecasting' published first in German in 1948 but later also in English translation was, apart from his daily weather charts, perhaps his greatest single achievement. His approach was fully three-dimensional from the beginning and his 'divergence theory' of synoptic development, already foreshadowed in the thirties drew much attention. His early upper air interests as a synoptic meteorologist and dynamical climatologist extended naturally into the higher levels of the stratosphere reached by sounding balloons (work in which his own station became extremely successful) and in 1952 he was the first to draw attention to what he called the explosive warming of the stratosphere in 1951-52. This discovery set in train a whole branch of stratospheric research familiarized everywhere by the name 'stratospheric sudden warming'.

The decision recently announced by the Executive Committee of the World Meteorological Organization to make the award of the Fifteenth International Meteorological Organization Prize to Professor Scherhag posthumously will be universally applauded as according to a leading

scientist the kind of recognition which, as a world meteorologist in every sense, he would so much have valued.

R. C. SUTCLIFFE

REVIEWS

Boundary-layer meteorology. Volume 1, No. 1, March 1970, edited by R. E. Munn. 240 mm × 160 mm, pp. 111, illus., D. Reidel Publishing Company, P.O. Box 17, Dordrecht - Holland, 1970. Price: Dfl. 140 per volume of 4 issues (reduced rate for private subscription.)

One result of the increasing, and seemingly inevitable, specialization within science is the introduction of journals which no longer attempt to cater for one of the major branches but for only one part of that branch. 'Boundary-layer meteorology' is one of the first in the meteorological sphere to devote itself in this way. Furthermore it aims to provide an opportunity for dialogue between two disciplines by inviting papers not only on the physical aspects of the boundary layer — the strictly meteorological problems — but also on the biological processes which interrelate with them. Examples of these are concerned with the transfers of heat, water and carbon dioxide from plants and animals on a time-scale which enables the interaction with the physical atmosphere to be explored and documented.

Whether this desirable bringing together of the two disciplines will work remains to be seen. The first number carries papers entirely devoted to the physical side, and it must be the target of the impressive editorial board to encourage writers on the biological side to come forward to achieve the right balance. If it succeeds in this, and at the same time can maintain a high standard in the quality of the papers it accepts, then it will perform a very useful function.

The journal is very pleasingly printed and produced.

F. B. SMITH

The chemical physics of ice, by N. H. Fletcher. 220 mm × 140 mm, pp. x + 271, illus., Cambridge University Press, Bentley House, 200 Euston Rd, London NW1, 1970. Price: £4.

Readers of Professor Fletcher's book on *The physics of rainclouds* will be pleased to find in this new book the characteristically lucid style and clear exposition with which they are already familiar.

Although it is clear that this book is sufficiently comprehensive to become a standard reference for all who are concerned with the physics of ice, it is not — and, indeed, is not intended to be — an encyclopedic collection of the data on ice. Rather, it is, as the author says in his preface, a book about chemical physics as well as ice. It was written with two classes of readers in mind. Firstly, there are advanced students who may benefit from applying their knowledge of general physical and chemical principles to a detailed study of one material — ice. Secondly, there are readers — like some meteorologists and cloud physicists — who already have a special interest in certain aspects of the subject, but who would like to have an up-to-date survey of the whole field.

Accordingly, Professor Fletcher begins with a detailed description of the individual water molecule. Using this picture as a basis, he then goes on to consider the structures of various forms of ice and the transition between the liquid and the solid state. In the final chapters, he uses this knowledge, together with the information which we now have about structural defects in ice, to discuss the thermal, mechanical and electrical properties of ice. The text is supported throughout by numerous references to published scientific papers so that the reader can pursue his studies further if he so wishes.

If there is a weakness in this book, it is perhaps in the question of the readership for whom it is designed. Firstly, it seems unlikely that many students who do not have a definite research interest in some aspect of ice will want to spend as much time on a single substance as this book implies. On the other hand, I suspect that many research workers will be disappointed to find the treatment of their own particular interest rather uncritical and will wonder whether other parts of the book suffer in the same way. However, this is probably an inevitable characteristic of a book of this size and scope.

In conclusion, the reviewer has no hesitation in recommending this book wholeheartedly to all who are seriously interested in the chemical physics of ice.

J. T. BARTLETT

551-578.466

NOTES AND NEWS

The following note appeared in *Symon's Rainfall Circular* for March 1865. This monthly circular was superseded in January 1866 by *Symon's Meteorological Magazine*, the forerunner of the present *Meteorological Magazine*. Reports of snow rollers have been given in the issues for June 1968 (p. 192), November 1968 (p. 350) and December 1969 (p. 387).

SNOW ROLLERS

In last month's circular a description of these singular formations was promised and in fulfilment thereof, the following statement has been drawn up from several communications forwarded by the observer (Rev. C. Clouston), who first drew attention to them in 1847, by a note in the *Philosophical Magazine*. They have hitherto only been observed in the vicinity of Sandwick Manse, and even there only on four occasions, viz. — Feb. 11, 1847, March 5, 1862, Feb. 18, and (a few) March 26, 1865, — a combination of the following conditions being necessary: (1) a recent fall of loose snow-flakes in calm weather; (2) temperature near 32° to give adhesion to the snow without thawing it; (3) a good brisk wind rising after the fall. Under these conditions, the snow ripples up, as it were, and the ripples breaking into sections, the wind rolls each in its own path, until, just like a schoolboy's snow-ball, they rapidly increase in size, and have been found 3½ feet long, and 7 feet in circumference; while others are not as many inches. "On examination, they are all found to be cylindrical, like hollow-fluted rollers, or ladies' swandown muffs, of which the smaller ones much remind me, from their lightness and purity. The centre is not quite hollow, but in all there is a deep

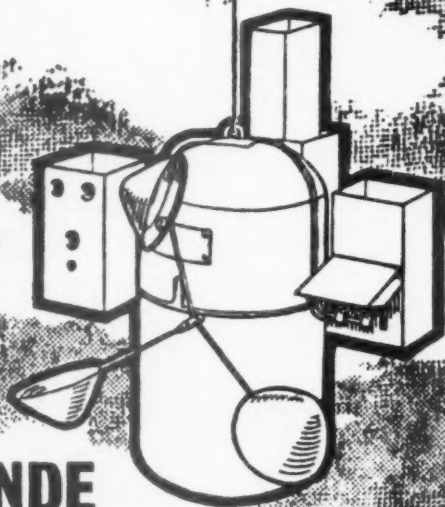
conical cavity at each end, and in many there is a small opening through which one can see." Their density seems to be about one-ninth that of water, as one 3 feet long, and $6\frac{1}{2}$ feet in circumference weighed 64lbs. Their number is variable; in 1847, 133 were counted in one acre, and nearly 400 acres were covered with them. It may be well to add that the manse stands near the top of a very gentle slope, rising from the sea, whence it is distant about two miles; the coast which is generally fringed with cliffs of considerable height, drops to, and even below the level of the sea, west of the manse, and allows the sea to form an arm, running nearly up to the manse grounds. There is no high ground in the neighbourhood, but gentle undulations in all directions. I cannot help thinking that the briny breeze which thwarts all attempts at growth on the part of trees in the open country, produces these beautiful snow rollers, though the relative influence of the wind and the salt remains to be investigated.

CORRECTION

Meteorological Magazine, December 1970, p. 363, 8th line, U.D.C. to read :
551.509.324.2:551.577



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